

## Math 1320: Compound Interest

**What is compound interest?** Many of us have savings accounts. Each month, the account may reflect a credit, called a dividend. That dividend is the compound interest our money accrues. Compound interest is interest that is computed on an original investment as well as on any accumulated interest.

**What questions may I be asked about compound interest?** The most appropriate application of compound interest is to help decide between different investment opportunities. There are many factors to consider, not only the interest rate, but also how often the interest is compounded (e.g. quarterly, monthly, continuously), when determining the maximum financial benefit. Some banks may compound interest continuously. In this case, the number of compounding periods increases exponentially.

The table below gives the formulas for the two different types of interest:

Interest for $n$ Compounding Periods per Years	Continuous Compounding Interest
$A = P\left(1 + \frac{r}{n}\right)^{nt}$	$A = Pe^{rt}$
<ul style="list-style-type: none"><li>• <math>A</math>: the balance (amount of money in account after interest is accrued)</li><li>• <math>P</math>: principal amount (amount of money initially invested)</li><li>• <math>r</math>: annual interest rate (in decimal form)</li><li>• <math>n</math>: # of compounding periods per year [e.g. monthly <math>n = 12</math>, quarterly <math>n = 4</math>]</li><li>• <math>t</math>: # of years</li></ul>	

For the following problems, identify the values of  $A$  and  $P$  we would use for computing compound interest. Is the interest compounded periodically or continuously? If the interest is compounded periodically, what is the value of  $n$ ?

1. You open a savings account and deposit \$250. The account earns 1.5 % monthly. How much money is in the account after 10 years?

[ $A = A$  since it is unknown,  $P = 250$ , Compounded Periodically with  $n = 12$ ]

2. Your parents invested \$500 in a savings account for you in 2010. The savings account is set to compound quarterly. In 2022, the account had a balance of \$7,800. What is the annual interest rate of the account?

[ $A = 7,800$ ,  $P = 500$ , Compounded Periodically with  $n = 4$ ]

3. You invested \$1500 in a CD that pays an annual interest rate of 4.3 % and compounds continuously. What is the value of the CD after 6 years?

[ $A = A$  since it is unknown,  $P = 1500$ , Compounded Continuously]

### Example 1. Choosing between Accounts

You decide to invest \$4,500 for 8 years and you have a choice between two different banks. The first bank offers a savings account that pays 1.20% per year, compounded monthly. The second bank offers a savings account that pays 1.15% per year, compounded continuously. Which bank offers the better investment?

**Solution:** The better investment is the one with the greater balance after 8 years. First we should identify the values of  $A, P, r, n, t$ .

- $A = A$  (unknown)
- $P = 4,500$
- $t = 8$

**Bank 1:** Interest is compounded monthly at 1.20% per year.

- $r = 0.012$
- $n = 12$

We need the formula:

$$A = P\left(1 + \frac{r}{n}\right)^{nt} = 4500\left(1 + \frac{0.012}{12}\right)^{12 \cdot 8} \approx \$4,953.18$$

The balance in the account for Bank 1 after 8 years is \$4,953.18

**Bank 2:** Interest is compounded continuously at 1.15% per year.

- $r = 0.0115$

We need the formula:

$$A = Pe^{rt} = 4500e^{0.0115 \cdot 8} \approx \$4,933.64$$

The balance in the account for Bank 2 after 8 years is \$4,933.64

The savings account at Bank 1 has a larger balance after 8 years. Therefore the savings account at Bank 1 with an interest rate of 1.20% per year, compounded monthly is the better investment option.